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journal homepage: www.elsevier.com/locate/amlConvergence result for the thermoelasticity of type III[☆]Yan Liu^{a,*}, Yiwu Lin^a, Yuanfei Li^b^a Department of Applied Mathematics, Guangdong University of Finance, Guangzhou 510521, PR China^b Department of Accounting, Huashang College, Guangdong University of Business Studies, Guangzhou 511300, PR China

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ABSTRACT

The convergence result for the thermoelasticity of type III defined on a semi-infinite cylindrical region is studied. We prove the convergence result for the thermal conductivity b .

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1. Introduction

Studies about the concept of structural stability have been gaining much impetus in recent years. We may recall the book of Ames and Straughan [1] and the monograph of Straughan [2] (also see [3–6] and the papers cited therein). In structural stability the emphasis is on continuous dependence (convergence result) on changes in the model itself rather than on the initial data. This means changes in coefficients in the partial differential equations and changes in the type of equations and may be reflected physically by changes in constitutive parameters. In addition, the inevitable error that arises in both numerical computation and the physical measurement of data can exist. It is relevant to know the magnitude of the effect of such errors on the solution. Here, we consider the thermoelastic theory proposed in the work of Green and Naghdi [7,8]. The governing equations of the linear theory of thermoelasticity of type III are

$$\rho \ddot{u}_i = \mu \Delta u_i + (\lambda + \mu) u_{j,ji} - \beta \theta_{,i}, \quad (1.1)$$

$$c \ddot{\alpha} = -\beta \dot{u}_{i,i} + k \Delta \alpha + b \Delta \theta. \quad (1.2)$$

Eq. (1.2) is a modification of the equation originally derived by Green and Naghdi. Here u_i is the displacement, the constant ρ is the density of the considered medium, θ is the temperature, α is a variable that satisfies $\dot{\alpha} = \theta$, λ and μ are the Lamé constants and we assume that they satisfy $\mu > 0$ and $\mu + \lambda > 0$. β is the coupling parameter and is related to the thermal expansion coefficient, $b > 0$ is the thermal conductivity, $c > 0$ is the specific heat and $k > 0$ is a parameter which is typical on the theories of type II and III. On a macroscopic scale the scalar α is regarded as representing some “mean” thermal displacement magnitude, and for brevity is referred to as thermal displacement. Its presence in some sense introduces a “thermal memory” and enhances heat propagation as a thermal displacement wave (both types II and III theories for heat flow in a stationary rigid solid accommodate finite wave speed). In [9,6,10], the authors have gained some results on the structural stability results in a bounded domain Ω . In the present paper, we can obtain the convergence result with respect

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to the heat conduction b in an unbounded domain. It is possible to investigate the effect of perturbation due to parameter k , the arguments being similar to which we employ. The authors are unaware of such results for thermoelasticity of type III in an unbounded domain.

Our attention is focused on the initial-boundary problems (1.1)–(1.2) in the space–time region $R \times (0, \infty)$, where $R = \{(x_1, x_2, x_3) | x_1 > 0, (x_2, x_3) \in D\}$, the arbitrary cross section D being a bounded simply-connected region in the (x_2, x_3) -plane with piecewise smooth boundary ∂D . We also use the notations

$$R_z = \{(x_1, x_2, x_3) | x_1 > z \geq 0, (x_2, x_3) \in D\}, \quad D_z = \{(x_1, x_2, x_3) | x_1 = z \geq 0, (x_2, x_3) \in D\}$$

for fixed z . Thus in particular $R_0 = R$, $D_0 = D$.

The lateral sides of the cylinder are constrained to have zero displacement and α . Thus, we have

$$u_i = 0, \quad \alpha = 0 \quad \text{on } \partial D \times (0, \infty), \quad t > 0. \quad (1.3)$$

We impose boundary conditions on the finite end of the cylinder. Thus, we take as assumptions

$$u_i(0, x_2, x_3, t) = f_i(x_2, x_3, t), \quad (x_2, x_3) \in D, \quad t > 0, \quad (1.4)$$

$$\alpha(0, x_2, x_3, t) = g(x_2, x_3, t), \quad (x_2, x_3) \in D, \quad t > 0. \quad (1.5)$$

To the system of field equations, we adjoin the initial conditions,

$$u(\mathbf{x}, 0) = \dot{u}(\mathbf{x}, 0) = \theta(\mathbf{x}, 0) = \alpha(\mathbf{x}, 0) = 0. \quad (1.6)$$

$$\dot{u}_i, u_{i,j}, \alpha_{,j}, \theta, \theta_{,i} \rightarrow 0 \quad (\text{uniformly in } x_2, x_3) \text{ as } x_1 \rightarrow \infty. \quad (1.7)$$

In the present paper, the comma is used to indicate partial differentiation, the differentiation with respect to the coordinate x_k is denoted as k ; thus $u_{,i}$ denotes $\frac{\partial u}{\partial x_i}$, and \dot{u} denotes $\frac{\partial u}{\partial t}$. The usual summation convention is employed with repeated Latin subscripts i summed from 1 to 3, and with repeated Greek subscripts α summed from 2 to 3. For example, $u_{i,i} = \sum_{i=1}^3 \frac{\partial u_i}{\partial x_i}$ and $u_{\alpha,\alpha} = \sum_{\alpha=2}^3 \frac{\partial u_\alpha}{\partial x_\alpha}$.

2. Convergence result as the parameter $b \rightarrow 0$

In this section, we investigate the convergence result on the parameter b . It would be possible to investigate the effect of perturbation for the other parameters, the arguments being similar to which we employ.

We denote by (v_i, ϕ) the solution of (1.1)–(1.7) with $b = 0$. If we set

$$w_i = u_i - v_i, \quad \pi = \alpha - \phi, \quad (2.1)$$

then we note that (w_i, π) satisfies the following initial and boundary problems

$$\rho \ddot{w}_i = \mu \Delta w_i + (\lambda + \mu) w_{j,i} - \beta \dot{\pi}_{,i} \quad \text{in } R \times \{t > 0\}, \quad (2.2)$$

$$c \ddot{\pi} = -\beta \dot{w}_{i,i} + k \Delta \pi + b \Delta \dot{\pi} + b \Delta \dot{\phi} \quad \text{in } R \times \{t > 0\}, \quad (2.3)$$

$$w(\mathbf{x}, 0) = \dot{w}(\mathbf{x}, 0) = \pi(\mathbf{x}, 0) = \dot{\pi}(\mathbf{x}, 0) = 0 \quad \text{in } R \times \{t = 0\}, \quad (2.4)$$

$$w_i = \pi = \dot{w}_i = \dot{\pi} = 0 \quad \text{on } \partial D \times \{t \geq 0\}, \quad (2.5)$$

$$w_i = \pi = \dot{w}_i = \dot{\pi} = 0 \quad \text{on } D_0 \times \{t > 0\}. \quad (2.6)$$

We define the energy

$$\begin{aligned} F(z, t) &= - \int_0^t \int_{D_z} (t-s) (\mu u_{i,1} \dot{u}_i + (\lambda + \mu) u_{j,j} \dot{u}_1 + \beta \dot{u}_1 \dot{\alpha} + k \alpha_{,1} \dot{\alpha} + b \theta_{,1} \dot{\theta}) dA ds \\ &= \frac{\rho}{2} \int_0^t \int_z^\infty \int_{D_\eta} \dot{u}_i \dot{u}_i dA d\eta ds + \frac{\mu}{2} \int_0^t \int_z^\infty \int_{D_\eta} u_{i,j} u_{i,j} dA d\eta ds \\ &\quad + \frac{\lambda + \mu}{2} \int_0^t \int_z^\infty \int_{D_\eta} u_{i,i} u_{j,j} dA d\eta ds + \frac{k}{2} \int_0^t \int_z^\infty \int_{D_\eta} \alpha_{,j} \alpha_{,j} dA d\eta ds \\ &\quad + \frac{c}{2} \int_0^t \int_z^\infty \int_{D_\eta} \dot{\alpha}^2 dA d\eta ds + b \int_0^t \int_z^\infty \int_{D_\eta} (t-s) \theta_{,i} \dot{\theta}_{,i} dA d\eta ds. \end{aligned} \quad (2.7)$$

We assume $\ddot{v} = \ddot{\phi} = 0$ in $R \times \{t = 0\}$, and define

$$\begin{aligned} \tilde{F}(z, t) &= - \int_0^t \int_{D_z} (t-s) (\mu \dot{v}_{i,1} \ddot{v}_i + (\lambda + \mu) \dot{v}_{j,j} \ddot{v}_1 + \beta \ddot{v}_1 \dot{\phi} + k \dot{\phi}_{,1} \ddot{\phi}) dA d\eta ds \\ &= \frac{\rho}{2} \int_0^t \int_z^\infty \int_{D_\eta} \ddot{v}_i \ddot{v}_i dA d\eta ds + \frac{\mu}{2} \int_0^t \int_z^\infty \int_{D_\eta} \dot{v}_{i,j} \dot{v}_{i,j} dA d\eta ds \\ &\quad + \frac{\lambda + \mu}{2} \int_0^t \int_z^\infty \int_{D_\eta} \dot{v}_{i,i} \dot{v}_{j,j} dA d\eta ds + \frac{k}{2} \int_0^t \int_z^\infty \int_{D_\eta} \dot{\phi}_{,j} \dot{\phi}_{,j} dA d\eta ds + \frac{c}{2} \int_0^t \int_z^\infty \int_{D_\eta} \ddot{\phi}^2 dA d\eta ds. \end{aligned} \quad (2.8)$$

From (2.7) and (2.8), we obtain

$$F(z, t) \leq -\left(k_1 t + \frac{1}{2}\right) \frac{\partial F(z, t)}{\partial z},$$

and

$$\tilde{F}(z, t) \leq -\left(\tilde{k}_1 t + \frac{1}{2}\right) \frac{\partial \tilde{F}(z, t)}{\partial z}.$$

Integrating them yield

$$F(z, t) \leq F(0, t) e^{-\frac{1}{k_1 t + \frac{1}{2}} z}, \quad (2.9)$$

and

$$\tilde{F}(z, t) \leq \tilde{F}(0, t) e^{-\frac{1}{\tilde{k}_1 t + \frac{1}{2}} z}, \quad (2.10)$$

where $k_1 = \max\{\frac{\lambda+2\mu+|\beta|}{\rho}, \frac{\lambda+2\mu}{\mu}, 1, \frac{|\beta|+k+b}{c}\}$, $\tilde{k}_1 = \max\{\frac{\lambda+2\mu+|\beta|}{\rho}, \frac{\lambda+2\mu}{\mu}, 1, \frac{|\beta|+k}{c}\}$.

We define a function $\Phi(z, t)$ for nonnegative z and t by

$$\begin{aligned} \Phi(z, t) = & \frac{\rho}{2} \int_0^t \int_z^\infty \int_{D_\eta} \dot{w}_i \dot{w}_i dA d\eta ds + \frac{\mu}{2} \int_0^t \int_z^\infty \int_{D_\eta} w_{i,j} w_{i,j} dA d\eta ds \\ & + \frac{\lambda + \mu}{2} \int_0^t \int_z^\infty \int_{D_\eta} w_{i,i} w_{j,j} dA d\eta ds + \frac{k}{2} \int_0^t \int_z^\infty \int_{D_\eta} \pi_j \pi_j dA d\eta ds \\ & + \frac{c}{2} \int_0^t \int_z^\infty \int_{D_\eta} \dot{\pi}^2 dA d\eta ds + b \int_0^t \int_z^\infty \int_{D_\eta} (t-s) \dot{\pi}_i \dot{\pi}_i dA d\eta ds. \end{aligned}$$

Using (2.9) and (2.10), we can easily get

$$\begin{aligned} \Psi(z, t) = & \int_z^\infty \Phi(\xi, t) d\xi \\ = & \frac{\rho}{2} \int_0^t \int_{R_z} (\xi - z) \dot{w}_i \dot{w}_i dx ds + \frac{\mu}{2} \int_0^t \int_{R_z} (\xi - z) w_{i,j} w_{i,j} dx ds \\ & + \frac{\lambda + \mu}{2} \int_0^t \int_{R_z} (\xi - z) w_{i,i} w_{j,j} dx ds + \frac{k}{2} \int_0^t \int_{R_z} (\xi - z) \pi_j \pi_j dx ds \\ & + \frac{c}{2} \int_0^t \int_{R_z} (\xi - z) \dot{\pi}^2 dx ds + b \int_0^t \int_{R_z} (\xi - z) (t-s) \dot{\pi}_i \dot{\pi}_i dx ds, \end{aligned} \quad (2.11)$$

which we rewrite as

$$\begin{aligned} \Psi(z, t) = & -\frac{\rho}{2} \int_0^t \int_{R_z} (t-s)_{,s} (\xi - z) \dot{w}_i \dot{w}_i dx ds - \frac{\mu}{2} \int_0^t \int_{R_z} (t-s)_{,s} (\xi - z) w_{i,j} w_{i,j} dx ds \\ & - \frac{\lambda + \mu}{2} \int_0^t \int_{R_z} (t-s)_{,s} (\xi - z) w_{i,i} w_{j,j} dx ds - \frac{k}{2} \int_0^t \int_{R_z} (t-s)_{,s} (\xi - z) \pi_j \pi_j dx ds \\ & - \frac{c}{2} \int_0^t \int_{R_z} (t-s)_{,s} (\xi - z) \dot{\pi}^2 dx ds + b \int_0^t \int_{R_z} (\xi - z) (t-s) \dot{\pi}_i \dot{\pi}_i dx ds. \end{aligned} \quad (2.12)$$

Repeated integration by parts leads to

$$\begin{aligned} \Psi(z, t) = & -\mu \int_0^t \int_{R_z} (t-s) \dot{w}_i w_{i,1} dx ds - (\lambda + \mu) \int_0^t \int_{R_z} (t-s) \dot{w}_1 w_{i,i} dx ds + \beta \int_0^t \int_{R_z} (t-s) \dot{\pi} \dot{w}_1 dx ds \\ & - \int_0^t \int_{R_z} (t-s) (\xi - z) b \dot{\pi}_j \dot{\phi}_j dx ds - \int_0^t \int_{R_z} (t-s) b \dot{\pi} \dot{\phi}_1 dx ds - k \int_0^t \int_{R_z} (t-s) \dot{\pi} \pi_1 dx ds \\ & - b \int_0^t \int_{R_z} (t-s) \dot{\pi} \dot{\pi}_1 dx ds. \end{aligned} \quad (2.13)$$

Using Schwarz's inequality, we obtain

$$\begin{aligned}\Psi(z, t) \leq & \frac{\mu}{2}t \int_0^t \int_{R_z} \dot{w}_i \dot{w}_i dx ds + \frac{\mu}{2}t \int_0^t \int_{R_z} w_{i,j} w_{i,j} dx ds + \frac{\lambda + \mu}{2}t \int_0^t \int_{R_z} \dot{w}_i \dot{w}_i dx ds \\ & + \frac{\lambda + \mu}{2}t \int_0^t \int_{R_z} w_{i,i} w_{j,j} dx ds + \frac{|\beta|}{2}t \int_0^t \int_{R_z} (\dot{\pi})^2 dx ds + \frac{|\beta|}{2}t \int_0^t \int_{R_z} \dot{w}_i \dot{w}_i dx ds \\ & + \frac{\epsilon_1 b}{2} \int_0^t \int_{R_z} (t-s)(\xi-z) \dot{\pi}_{,i} \dot{\pi}_{,i} dx ds + \frac{b}{2\epsilon_1} \int_0^t \int_{R_z} (t-s)(\xi-z) \dot{\phi}_{,i} \dot{\phi}_{,i} dx ds \\ & + \frac{bt}{2} \int_0^t \int_{R_z} (\dot{\pi})^2 dx ds + \frac{b}{2} \int_0^t \int_{R_z} (t-s) \dot{\phi}_{,i} \dot{\phi}_{,i} dx ds + \frac{kt}{2} \int_0^t \int_{R_z} (\dot{\pi})^2 dx ds \\ & + \frac{kt}{2} \int_0^t \int_{R_z} \pi_{,i} \pi_{,i} dx ds + \frac{bt}{2} \int_0^t \int_{R_z} (\dot{\pi})^2 dx ds + \frac{b}{2} \int_0^t \int_{R_z} (t-s) \dot{\pi}_{,i} \dot{\pi}_{,i} dx ds,\end{aligned}\quad (2.14)$$

where ϵ_1 is an arbitrary positive constant.

Using (2.10), we have

$$\frac{b}{2} \int_0^t \int_{R_z} (t-s) \dot{\phi}_{,i} \dot{\phi}_{,i} dx ds \leq \frac{bt}{k} \tilde{F}(0, t) e^{-\frac{1}{k_1 t + \frac{1}{2}} z}, \quad (2.15)$$

$$\frac{b}{2\epsilon_1} \int_0^t \int_{R_z} (t-s)(\xi-z) \dot{\phi}_{,i} \dot{\phi}_{,i} dx ds \leq \frac{b \left(\tilde{k}_1 t + \frac{1}{2} \right)}{2\epsilon_1 k} \tilde{F}(0, t) e^{-\frac{1}{k_1 t + \frac{1}{2}} z}. \quad (2.16)$$

If we choose ϵ_1 small enough such that $\frac{\epsilon_1 b}{2} < 1$ and combine (2.10), (2.14), (2.15), and (2.16), we get

$$\Psi(z, t) \leq \left(\frac{k_2 t + \frac{1}{2}}{1 - \frac{\epsilon_1 b}{2}} \right) \Phi(z, t) + \left(\frac{\left(\tilde{k}_1 t + \frac{1}{2} \right)}{\frac{2\epsilon_1 k}{1 - \frac{\epsilon_1 b}{2}} + \frac{t}{k}} \right) b \tilde{F}(0, t) e^{-\frac{1}{k_1 t + \frac{1}{2}} z}, \quad (2.17)$$

where $k_2 = \max\{\frac{2\mu+\lambda+|\beta|}{\rho}, 1, \frac{2b+k}{c}\}$.

For any fixed $t_1 > 0$, we define $k_3 = \frac{k_2 t_1 + \frac{1}{2}}{1 - \frac{\epsilon_1 b}{2}}$, $k_4 = \frac{(\tilde{k}_1 t_1 + \frac{1}{2}) + \frac{t_1}{k}}{\frac{2\epsilon_1 k}{1 - \frac{\epsilon_1 b}{2}} + \frac{t_1}{k}}$, $k_5 = \frac{1}{k_1 t_1 + \frac{1}{2}}$, to obtain

$$\Psi(z, t_1) \leq k_3 \left(-\frac{\partial \Psi(z, t_1)}{\partial z} \right) + k_4 b \tilde{F}(0, t) e^{-k_5 z}. \quad (2.18)$$

We recall from (2.10) that $\frac{\partial \Phi(z, t_1)}{\partial z} \leq 0$, so that from (2.18) we easily obtain

$$\frac{\partial \Phi(z, t_1)}{\partial z} + \int_z^\infty \Phi(\xi, t_1) d\xi \leq k_3 \Phi(z, t_1) + k_4 b \tilde{F}(0, t) e^{-k_5 z}. \quad (2.19)$$

The following discussions are motivated by [11,12]. It remains to integrate (2.19). Let

$$\Lambda(z, t_1) = E(z, t_1) + \alpha \int_z^\infty e^{k_3(\xi-z)} E(\xi, t_1) d\xi, \quad (2.20)$$

where $E(z, t_1)$ is defined as

$$E(z, t_1) = e^{-k_3 z} \Phi(z, t_1) \quad (2.21)$$

and α is an arbitrary constant to be chosen later.

It is easy to show that (2.19) may be rewritten as

$$\frac{\partial \Lambda(z, t_1)}{\partial z} + \alpha \Lambda(z, t_1) \leq k_4 b \tilde{F}(0, t_1) e^{-k_3 z} e^{-k_5 z} \quad (2.22)$$

provided α satisfies the quadratic equation

$$\alpha^2 - k_3 \alpha - 1 = 0. \quad (2.23)$$

We make the choice of $\alpha = \alpha_0$, where

$$\alpha_0 = \frac{k_3 + \sqrt{k_3^2 + 4}}{2}. \quad (2.24)$$

For this choice of α , an integration of (2.22) yields the following two cases.

If $\alpha_0 - k_3 - k_5 \neq 0$, we get

$$\Lambda(z, t_1) \leq \left[\Lambda(0, t_1) - \frac{1}{\alpha_0 - k_3 - k_5} k_4 b \tilde{F}(0, t_1) \right] e^{-\alpha_0 z} + \frac{1}{\alpha_0 - k_3 - k_5} k_4 b \tilde{F}(0, t_1) e^{-(k_3 + k_5)z}. \quad (2.25)$$

From (2.25), we may deduce that

$$\Phi(z, t_1) \leq \left[\Lambda(0, t_1) - \frac{1}{\alpha_0 - k_3 - k_5} k_4 b \tilde{F}(0, t_1) \right] e^{-(\alpha_0 - k_3)z} + \frac{1}{\alpha_0 - k_3 - k_5} k_4 b \tilde{F}(0, t_1) e^{-k_5 z}. \quad (2.26)$$

If $\alpha_0 - k_3 - k_5 = 0$, we get

$$\Lambda(z, t_1) \leq \Lambda(0, t_1) e^{-\alpha_0 z} + k_4 b z \tilde{F}(0, t_1) e^{-\alpha_0 z}. \quad (2.27)$$

From (2.27), we can easily get that

$$\Phi(z, t_1) \leq \Lambda(0, t_1) e^{-(\alpha_0 - k_3)z} + k_4 b z \tilde{F}(0, t_1) e^{-(\alpha_0 - k_3)z}. \quad (2.28)$$

In order to make inequalities (2.26) and (2.28) explicit, we need bound for $\Lambda(0, t_1)$. In view of (2.20) and (2.21), we only need to bound $\Phi(0, t_1)$.

Now multiplying (2.2) by $(t_1 - s)\dot{w}_i$ and integrating over $R \times (0, t_1)$, we obtain

$$\begin{aligned} & \frac{\rho}{2} \int_0^{t_1} \int_R \dot{w}_i \dot{w}_i dx ds + \frac{\mu}{2} \int_0^{t_1} \int_R w_{i,j} w_{i,j} dx ds + \frac{\lambda + \mu}{2} \int_0^{t_1} \int_R w_{i,i} w_{j,j} dx ds \\ &= -\mu \int_0^{t_1} \int_{D_0} (t_1 - s) w_{i,1} \dot{w}_i dA ds - (\lambda + \mu) \int_0^{t_1} \int_{D_0} (t_1 - s) w_{j,j} \dot{w}_1 dA ds - \beta \int_0^{t_1} \int_R (t_1 - s) \dot{w}_i \dot{\pi}_{,i} dx ds. \end{aligned} \quad (2.29)$$

Multiplying (2.3) by $(t_1 - s)\dot{\pi}$ and integrating over $R \times (0, t_1)$ we obtain

$$\begin{aligned} & \frac{k}{2} \int_0^{t_1} \int_R \pi_{,j} \pi_{,j} dx ds + \frac{c}{2} \int_0^{t_1} \int_R \dot{\pi}^2 dx ds + b \int_0^{t_1} \int_R (t - s) \dot{\pi}_{,i} \dot{\pi}_{,i} dx ds \\ &= -\beta \int_0^{t_1} \int_R (t_1 - s) \dot{w}_{i,i} \dot{\pi} dx ds - b \int_0^{t_1} \int_{D_0} (t_1 - s) \pi_{,1} \dot{\pi} dA ds + b \int_0^{t_1} \int_R \dot{\pi} \Delta \dot{\phi} dx ds. \end{aligned} \quad (2.30)$$

Combining (2.29) and (2.30), and using (2.6), we obtain

$$\begin{aligned} \Phi(0, t_1) &= b \int_0^{t_1} \int_R (t_1 - s) \dot{\pi} \Delta \dot{\phi} dx ds \\ &= -b \int_0^{t_1} \int_R (t_1 - s) \dot{\pi}_{,i} \dot{\phi}_{,i} dx ds - b \int_0^{t_1} \int_{D_0} (t_1 - s) \dot{\pi} \dot{\phi}_{,1} dx ds \\ &= -b \int_0^{t_1} \int_R (t_1 - s) \dot{\pi}_{,i} \dot{\phi}_{,i} dx ds. \end{aligned} \quad (2.31)$$

By Schwarz's inequality, we obtain

$$\Phi(0, t_1) \leq \sqrt{\frac{2bt_1}{k}} [\Phi(0, t_1) \tilde{F}(0, t_1)]^{\frac{1}{2}}.$$

Which results in

$$\Phi(0, t_1) \leq \frac{2bt_1}{k} \tilde{F}(0, t_1). \quad (2.32)$$

From (2.18) we have

$$\int_0^\infty \Phi(z, t_1) dz \leq k_3 \Phi(0, t_1) + k_4 b \tilde{F}(0, t_1) e^{-k_5 z}. \quad (2.33)$$

In view of (2.20), (2.21), (2.32) and (2.33) we obtain

$$\Lambda(0, t_1) \leq \left(\frac{2(k_3\alpha_0 + 1)}{k} \right) \tilde{F}(0, t_1) + k_4 b \alpha_0 \tilde{F}(0, t_1) e^{-k_5 z}. \quad (2.34)$$

Combining the above discussions, we can conclude when $\alpha_0 \neq k_3 + k_5$, we have

$$\Phi(z, t_1) \leq \tilde{F}(0, t_1) b \left\{ \left[\frac{2k_3 t_1}{k} k_4 e^{-k_5 z} e^{-(\alpha_0 - k_3)z} \right] + \frac{k_4}{\alpha_0 - k_3 - k_5} e^{-k_5 z} \right\}, \quad (2.35)$$

and when $\alpha_0 = k_3 + k_5$, we have

$$\Phi(z, t_1) \leq \tilde{F}(0, t_1) b e^{-k_5 z} \left\{ \frac{2k_3 t_1}{k} t_1 + k_4 e^{-k_5 z} + k_4 z \right\}. \quad (2.36)$$

Inequalities (2.35) and (2.36) exhibit not only exponential decay in z , but also show that the amplitude terms in (2.35) and (2.36) become small as $b \rightarrow 0$.

Summarizing all the above discussions, we can establish the following theorem.

Theorem 1. Let (u_i, α) and (v_i, ϕ) be the classical solutions of the thermoelasticity of type III for different values of $b > 0$ and $b = 0$ respectively, (w_i, π) be the difference of (u_i, α) and (v_i, ϕ) , the estimates (2.35) and (2.36) are satisfied.

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